AP Calculus
Summer Worksheet - Precalculus Review

Proficiency in each of the following areas is expected of all students taking AP calculus. Do your best to be proficient in all areas of this worksheet when you report to class at the beginning of the fall trimester. Feel free to use Chapter P of your calculus text or material from last year's precalculus book as a reference for help in addition to the examples that are provided. There will be time to cover questions on the first couple of days and then you will be quizzed on these skills.

I. **Simplify algebraic expressions** $\rightarrow$ Be able to rewrite algebraic expressions in different ways.

A. Rewrite each of the following as a power function $\rightarrow$ kx^p or as an exponential function $\rightarrow$ $a \cdot b^x$ where k, p, a and b are constants. Be able to identify the values for k, p, a and b.

i. $e^{(2x+1)}$

ii. $\dfrac{e^3}{e^{(-x+4)}}$

iii. $\dfrac{3\sqrt{x}}{4}$

iv. $\sqrt[3]{8x^{-5}} \cdot \left(5\sqrt{x}\right)^3$

v. $\dfrac{\frac{1}{3}}{2x^7}$

vi. $\dfrac{5^x}{-3^x}$

vii. $2^x \cdot 3^{(x-1)}$

viii. $\left(10e^x\right)^2$

Examples

1. $\dfrac{\sqrt[3]{x}}{5x^3} = \dfrac{1}{5} \cdot \dfrac{x^{1/3}}{x^3} = \frac{1}{5} x^{\left(\frac{1}{3}-3\right)} = \frac{1}{5} x^{-\frac{8}{3}}$

2. $2^{-n} = \dfrac{1}{2^n} = \dfrac{1^n}{2^n} = \left(\dfrac{1}{2}\right)^n = 1 \cdot \left(\dfrac{1}{2}\right)^n$

3. $\left(2e^{2x}\right)^2 = 4e^{4x} = 4 \cdot \left(e^4\right)^x \approx 4\left(54.60^x\right)$

B. Rewrite each quadratic expression in the form $a\left(x-h\right)^2 + k$ by completing the square.

i. $x^2 - 2x - 3$

ii. $6x - 2 - x^2$

iii. $3x^2 - 12x + 13$

Example

$$5x^2 + 30x - 10$$
$$5\left(x^2 + 6x + \underline{9}\right) - 10 - 45$$
$$5\left(x+3\right)^2 - 55$$

C. Rewrite as a sum or difference of power functions.

i. $\dfrac{6t^3 + 3t - 4}{3t^2}$

ii. $\dfrac{x^{-1/2} + \sqrt{x}}{x^2}$

Example

$$\frac{\sqrt{x}+3}{3\sqrt{x}}$$
$$\frac{x^{1/2}+3}{3x^{1/2}}$$
$$\frac{x^{1/2}}{3x^{1/2}} + \frac{3}{3x^{1/2}} = \frac{1}{3} + \frac{1}{x^{1/2}} = \frac{1}{3} + x^{-1/2}$$

D. Perform the given operation and simplify the expression as much as possible.

i. $\frac{10}{y-2}+\frac{3}{2-y}$

ii. $\frac{1}{e^{2x}}+\frac{1}{e^{x}}$

iii. $\frac{8}{3x^2-x-4}-\frac{9}{x+1}$

Example

$$\frac{a}{a^2-9}+\frac{2}{a-3}$$
$$\frac{a}{(a-3)(a+3)}+\frac{2}{a-3}$$
$$\frac{a+2(a+3)}{(a-3)(a+3)}$$
$$\frac{3a+6}{(a-3)(a+3)}=\frac{3(a+2)}{(a-3)(a+3)}$$

E. Use inverse operations to solve for the indicated variable.

i. $T=2\pi\sqrt{\frac{m}{g}}\,,\ m=?$

ii. $h=v_0t+\frac{1}{2}at^2,\ a=?$

iii. $S=\frac{rL-a}{r-1},\ r=?$

iv. $7e^2=e^{kt},\ k=?$

Examples

$$\frac{1}{2}P_0=P_0(0.8)^x,\ x=?$$
$$\frac{1}{2}=(0.8)^x$$
$$\ln\frac{1}{2}=\ln\left[(0.8)^x\right]$$
$$\ln 0.5=x\ln(0.8)$$
$$\frac{\ln 0.5}{\ln 0.8}=x$$

$$by-d=ay+c,\ y=?$$
$$by-ay=c+d$$
$$y(b-a)=c+d$$
$$y=\frac{c+d}{b-a}$$

II. **Simplify Trigonometric Expressions** → Simplify each of the following expressions completely using basic trigonometric identities, including the Pythagorean identities and the double angle identities

A. $\sin x+\cos x\tan x$

B. $\frac{\cos\theta}{\sec\theta-\tan\theta}$

C. $\sin^2 A\cdot\cos^2 A+\sin^4 A$

D. $\frac{\sin 2x}{1-\cos 2x}$

Examples

$$\frac{\sec\alpha}{\sin\alpha}-\frac{\sin\alpha}{\cos\alpha}$$
$$\frac{\frac{1}{\cos\alpha}}{\sin\alpha}-\frac{\sin\alpha}{\cos\alpha}$$
$$\frac{1}{\cos\alpha\sin\alpha}-\frac{\sin\alpha}{\cos\alpha}$$
$$\frac{1-\sin^2\alpha}{\cos\alpha\sin\alpha}$$
$$\frac{\cos^2\alpha}{\cos\alpha\sin\alpha}=\frac{\cos\alpha}{\sin\alpha}=\cot\alpha$$

$$\frac{\sin 2x}{1+\cos 2x}$$
$$\frac{2\sin x\cos x}{1+\left(2\cos^2 x-1\right)}$$
$$\frac{2\sin x\cos x}{2\cos^2 x}$$
$$\frac{\sin x}{\cos x}$$
$$\tan x$$

III. **More practice with trigonometric values**.

A. Periodically you should review the unit circle values so that you can retrieve one when it is necessary to do so. Determine each of the following values without using the calculator.

i. $\cos\dfrac{\pi}{2}$ ii. $\sin\dfrac{3\pi}{4}$ iii. $\csc\left(-\dfrac{\pi}{6}\right)$ iv. $\cot 0$

v. $\tan\dfrac{2\pi}{3}$ vi. $\csc\dfrac{3\pi}{2}$ vii. $\sin\dfrac{7\pi}{6}$ viii. $\cos(-\pi)$

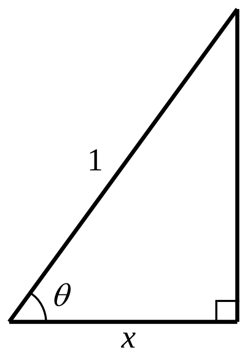
B. It is also important to be able to use right triangles to manipulate trigonometric values. Use the drawing to express each of the following in terms of x.

i. $\cos\theta$

ii. $\cos\left(\dfrac{\pi}{2}-\theta\right)$

iii. $\tan^2\theta$

iv. $\cos(2\theta)$



<u>Examples</u>

$$\csc\left(\frac{\pi}{2}-\theta\right)$$
$$\frac{1}{\sin\left(\frac{\pi}{2}-\theta\right)}$$
$$\frac{1}{\frac{x}{1}}=\frac{1}{x}$$

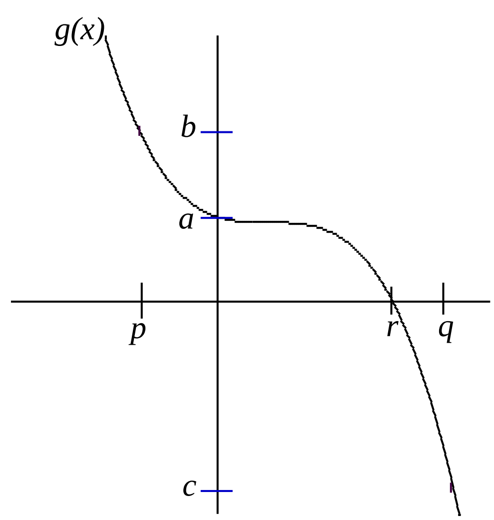
$$\sin(2\theta)$$
$$2\sin\theta\cos\theta$$
$$2\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right)$$
$$2x\sqrt{1-x^2}$$

IV. **Functions** $\rightarrow$ The ability to work with any type of function is crucial to success in calculus. This section will have you work with functions graphically and analytically, as well as test your knowledge of function notation.

A. Determine each of the following using the graph of $g(x)$. Each answer should be in terms of $a,\ b,\ c,\ p,\ q$ or r.

i. $g(q)=$_______ iii. $g(g(r))=$_______

ii. $-g(p)=$_______ vi. $g^{-1}(b)=$_______



B. Fill in each blank with $<,\ >$ or $=$.

i. $g(p)$____$g(0)$ iii. $g(p)-g(q)$____0

ii. $g^{-1}(c)$____$g^{-1}(0)$ vi. $g(0)\cdot g(q)$____0

C. Given function $f(x)$ such that $f(-2)=5$, complete each of the following statements.

i. One (x, y) coordinate of a point that must be on the graph of $f^{-1}(x)$ is ______________.

ii. If $f(x)$ is an odd function, then the (x, y) coordinates of 2 points that must be on the graph of $f(x)$ are ______________ and _________________.

iii. If $f(x)$ is an even function, then the (x, y) coordinates of 2 points that must be on the graph of $f(x)$ are ______________ and _________________.

D. Given the function $h(x)=\begin{cases} -2x-4, & x<-2 \\ -(x-1)^2-3, & x \geq 0 \end{cases}$, determine each of the following.

i. $h(-2)=$ ______ ii. $h(0)=$ ______ iii. $h(5)-h(-3)=$ ______

iv. Determine the domain of $h(x)$.

v. Determine the range of $h(x)$.

vi. If they were defined for all the real numbers, at what point(s) do the two pieces of the function $h(x)$ intersect. i.e. Without using the graphing capabilities of your calculator, solve the following system of equations $\begin{cases} y=-2x-4 \\ y=-(x-1)^2-3 \end{cases}$

E. Determine a formula for the inverse of each of the following functions.

i. $y=12(x-1)^3$

ii. $y=e^{(3x+1)}$

iii. $y=\ln\left(\frac{1}{4}x\right)-3$

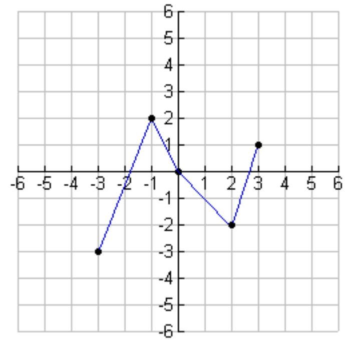
iv. $y=\frac{3+2x}{2-5x}$

Examples

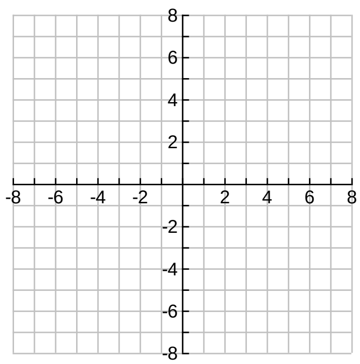
$$y=\ln(x-1)+2$$
$$x=\ln(y-1)+2$$
$$x-2=\ln(y-1)$$
$$e^{(x-2)}=y-1$$
$$e^{(x-2)}+1=y$$

$$y=\frac{x+3}{11-2x}$$
$$x=\frac{y+3}{11-2y}$$
$$x(11-2y)=y+3$$
$$11x-2xy=y+3$$
$$11x-3=2xy+y$$
$$11x-3=y(2x+1)$$
$$\frac{11x-3}{2x+1}=y$$

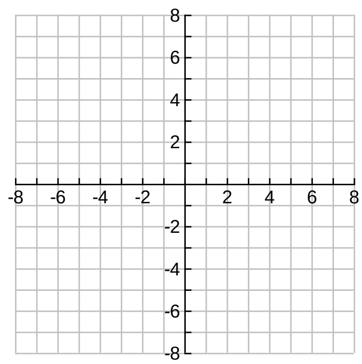
V. **Transformations of functions** $\rightarrow$ reading transformations accurately will help you to quickly visualize situations, especially when a calculator is not available. Given the graph of $f(x)$ at the right, graph each of the following on the axes provided.



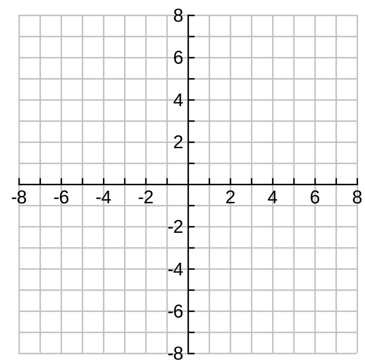
A. $f(x-2)$



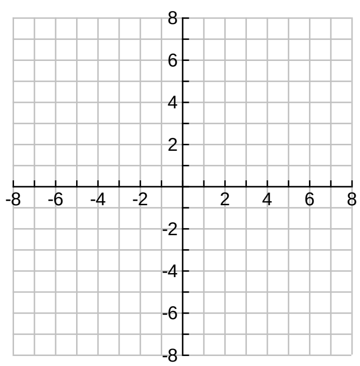
B. $-2 \cdot f(x)$



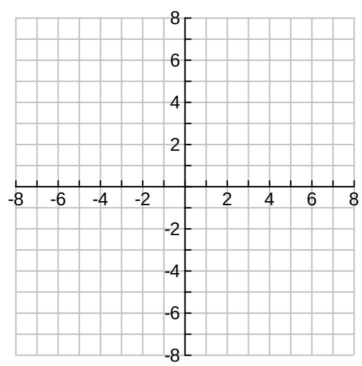
C. $f(2x)+3$



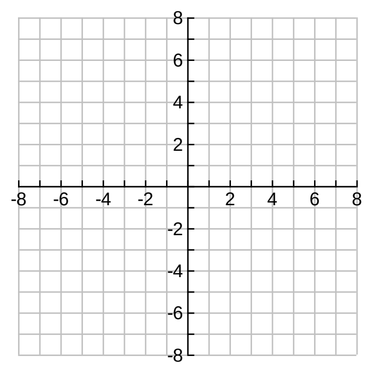
D. $|f(x)|$



E. $f(|x|)$



F. $f^{-1}(x)$



VI. **Linear functions and polynomials** $\rightarrow$ writing linear equations needs to be automatic in calculus.

A. A line has a slope of 3 and contains the point $(-2,-6)$.

i. Write the equation for this line.

ii. Write the equation for a line that is perpendicular to this line through the point $(-2,-6)$.

<u>Example</u>

Write the equation of a line that has a slope of $\frac{1}{2}$ and contains the point $(1,-7)$.

B. *Multiple Choice*: Determine which of the following is the most correct equation for the given polynomial that contains the point $(2,\ 12)$.

i. $\frac{1}{16}x(x-4)(x+3)(x-2)^2(x+1)$

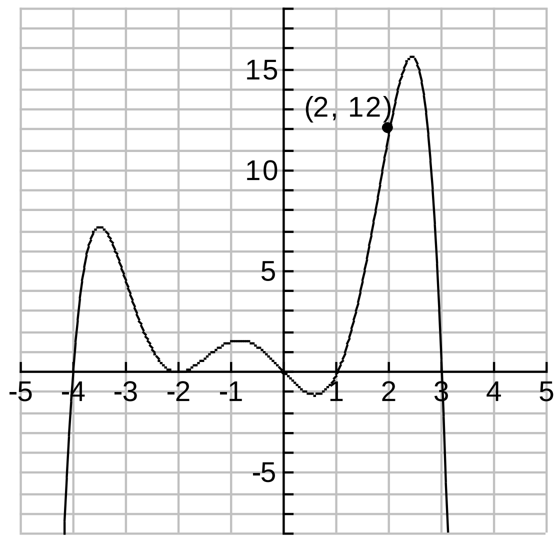
ii. $\frac{1}{16}x(x+4)(x-3)(x+2)^2(x-1)$

iii. $-\frac{1}{16}x(x+4)(x-3)(x+2)(x-1)$

iv. $-\frac{1}{16}x(x+4)(x-3)(x+2)^2(x-1)$

v. $\frac{1}{16}x(x-4)(x+3)(x-2)(x+1)$

vi. $-\frac{1}{16}(x+4)(x-3)(x+2)^2(x-1)$



C. Given the polynomial $f(x)=2x^2-x$, write simplified formulas for each of the following.

i. $f\left(\frac{1}{x}\right)$

ii. $f(x+h)$

iii. $f\left(f(x)\right)$

<u>Example</u>

Write a formula for $h(x+1)$ given $h(x)=\frac{x}{x-1}$

$$h(x+1)=\frac{x+1}{(x+1)-1}$$

$$h(x+1)=\frac{x+1}{x} \text{ or } \frac{x}{x}+\frac{1}{x}=1+\frac{1}{x}$$

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Answer Key

I. A. i. $e\left(e^2\right)^x$ or $e \cdot 7.389^x$

ii. $\frac{1}{e} \cdot e^x$

iii. $\frac{3}{4} x^{\frac{1}{2}}$

iv. $250x^{-\frac{1}{6}}$

v. $\frac{1}{6} x^{-7}$

vi. $-\left(\frac{5}{3}\right)^x = -1 \cdot \left(\frac{5}{3}\right)^x$

vii. $\frac{1}{3} \cdot 6^x$

viii. $100\left(e^2\right)^x$ or $100 \cdot 7.389^x$

B. i. $(x-1)^2 - 4$

ii. $-(x-3)^2 + 7$

iii. $3(x-2)^2 + 1$

C. i. $2t + t^{-1} - \frac{4}{3} t^{-2}$

ii. $x^{-\frac{5}{2}} + x^{-\frac{3}{2}}$

D. i. $\frac{7}{y-2}$

ii. $\frac{1+e^x}{e^{2x}}$

iii. $\frac{-27x+44}{(x+1)(3x-4)}$

E. i. $m = \frac{gT^2}{4\pi^2}$

ii. $\frac{2}{t^2}\left(h - v_0 t\right)$

iii. $r = \frac{S-a}{S-L}$

iv. $\frac{1}{t}(\ln 7 + 2)$

II. A. $2\sin x$

B. $1 + \sin\theta$

F. $\sin^2 A$

G. $\cot x$

III. A. i. 0

ii. $\frac{\sqrt{2}}{2}$

iii. -2

iv. undefined

v. $-\sqrt{3}$

vi. -1

vii. $-\frac{1}{2}$

viii. -1

B. i. $\frac{x}{1} = x$

ii. $\frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

iii. $\frac{1-x^2}{x^2}$

iv. $2x^2 - 1$

IV. A. i. c

ii. $-b$

iii. a

iv. p

B. i. $>$

ii. $>$

iii. $>$

iv. $<$

C. i. $(5,-2)$

ii. $(-2,\ 5)$ and $(2,\ -5)$

iii. $(-2,\ 5)$ and $(2,\ 5)$

D. i. undefined

ii. -4

iii. -21

iv. $(-\infty,-2)\cup[0,\infty)$

v. $(-\infty,-3]\cup(0,\infty)$

vi. $(0,-4)$ and $(4,-12)$

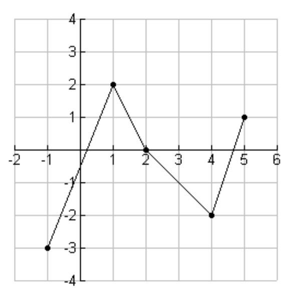
E. i. $\sqrt[3]{\dfrac{x}{12}}+1=y$

ii. $y=\dfrac{1}{3}(\ln x-1)$

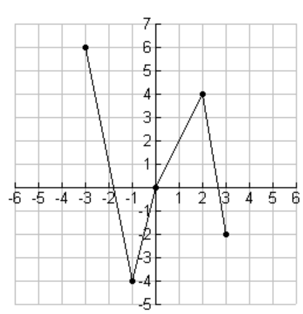
iii. $y=4e^{(x+3)}$

iv. $y=\dfrac{2x-3}{5x+2}$

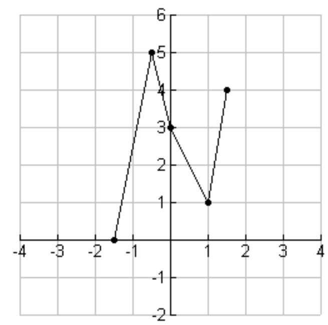
V. A. Shift right 2 units



B. Vertical stretch by a factor of 2 & reflect over *x*-axis.

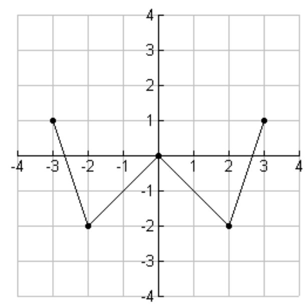


C. Horizontal compression by a factor of ½ & shift up 3 units

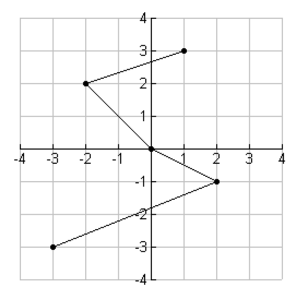


D. All *y*-values should be positive.

E. Graph should be symmetric with respect to the y-axis.



F. For each point switch the *x* & the *y* coordinates.



VI. A. i. $y=3(x+2)-6$

ii. $y=-\dfrac{1}{3}(x+2)-6$

B. iv is the correct choice.

C. i. $\dfrac{2-x}{x^2}$

ii. $2x^2+4hx-x+2h^2-h$ or $2x^2+(4h-1)x+2h^2-h$

iii. $8x^4-8x^3+x$